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THE SURFACE ROUGHNESS EFFECTS IN COMPUTATION OF THE TURBULENT BOUNDARY LAYER ON SLENDER SHIP-HULL

Si-Young Kim* and A.K. Lewkowicz**

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An improved version of an integral method for computing turbulent boundary layers on a slender ship-hull with auxiliary shape parameter and lag-entrainment concept is suggested to account more effectively for the surface roughness contribution to the friction resistance coefficient. Comparisons with wind-tunnel model test results show some significant improvements. The improved version is then employed to test the consequences of zonal hull roughening along with streamline verifying and modifying an approximate technique of scaling model-to-ship roughness effects upon the hull friction coefficient.

Key Words: Slender Ship-Hull, Turbulence, Auxiliary Shape Parameter, Wall Friction, Local-Roughness, Model-to-Ship Scaling, Roughness Function, Lag-entrainment.

1. INTRODUCTION

Ever increasing sophistication in prediction of ship friction resistance necessitates better than hitherto used ways of accounting for the hull roughness, the occurrence of which is almost always inevitable in real marine environment.

Ultimately, the continually increasing computer power will allow to include the effects directly in the CFD/N-S procedures treating them as supergrid features. However. until this is possible, the roughness effects can be conveniently modeled by means of a turbulent boundary layer(TBL) on wall friction. This approach has been adopted by amny and in the present marine context, for example, by Okuno and Lewkowicz (1987) and Hoekstra(1983).

This paper reports on some refinements carried out recently within the scheme used by Okuno and Lewkowicz (1987). More realistic relationships for both the streamwise wall friction with auxiliary shape parameter, accounting for the local and distributed surface roughness effects (the other method described by Nakato et al.(1984)), are proposed along with a replacement of the simple entrainment quation Head (1958)) in the Okuno and Lewkowicz (1987) by the physically stronger concept of 'lag-entrainment' invented initially by Green et al.(1977) for hydrofoils and modified by Das and Lewkowicz(1986) for plaiable rough surface.

The lag-entrainment equation had been used before by Lewkowicz and Das (1981), (1986) and Lewkowicz (1985) to predict two-dimensional TBLs with different pressure gradients on rough surfaces of marine nature (combination of solid and flexible roughness components). This was seen to have enhanced the predictive capabilities of the integral procedure. For which reason the present authors decided to incorporate the particular lag-entrainment method in three-dimensional TBLs predictions on the slender ship-hull with auxiliary shape parameter as formerly given in Okuno and Lewkowioz(1987).

*Marine Engineering Dept., National Fisheries University of Pusan, Pusan 608-737, Korea
**Mechanical Engineering Dept., The University of Liverpool, Liverpool, L69, U.K.

2. AUXILIARY SHAPE PARAMETER AND LAG-ENTRAINMENT FOR ROUGHNESS

Assuming that the Cole's law of the wall and the law of the wake hold true for both smooth and rough wall TBLs, a relation can be derived to link the change in shape parameter $H(=\delta^*/\theta)$, caused by the surface roughness, with the usual Clauser roughness function $\Delta U/u_o$ in :

$$\frac{U}{u_o}=2.44\ ln\left(\frac{hu_o}{\nu}\right)+5.0+2.44\Pi[2-\omega(\eta)]-\frac{\Delta U}{u_o} \quad (1)$$

where $u_o(=\sqrt{(\tau_o/\rho)})$ is the wall friction velocity, Π is the Cole's wake strength parameter, $\omega(\eta)$ is universal wake function as given by Lewkowicz(1982) in the form of $\omega(\eta)=2\eta^2(3-2\eta)-\Pi^{-1}\eta^2(1-3\eta+2\eta^2)$, and ν is the kinematic viscosity of fluid. From Eq. (1), the shape factor H is given by

$$H=1-\sqrt{\frac{C_f}{2}\frac{I_1}{I_2}} \quad (2)$$

where $C_f=2\tau_o/\rho U\infty^2$=wall friction coefficient, $I_1=2.44(\Pi+0.983)$, $I_2=5.95(1.486\Pi^2+3.176\Pi+2.009)(\Pi+0.983)^{-1}$, and $U\infty$ is the free-stream velocity outside the boundary layer.

The Clauser roughness function(1956) reflects the difference between the rough (subscript r) and smooth (subscript s) wall friction coefficients as the following equation :

$$\sqrt{\frac{2}{C_{fs}}}-\sqrt{\frac{2}{C_{fr}}}=\frac{\Delta U}{u_o} \quad (3)$$

Now writing $\omega=\sqrt{C_f/2}$, the above equation can be expressed as the following equation;

$$\frac{1}{\omega_s}-\frac{1}{\omega_r}=\frac{\Delta U}{u_o} \tag{4}$$

The Eq. (4) may be written as follows,

$$\omega_s-\omega_r=-\frac{\Delta U}{u_o}(\omega_s\cdot\omega_r) \tag{5}$$

and auxiliary shape parameter can be derived as the following equation:

$$\begin{aligned}\Delta H&=\sqrt{\frac{C_{fs}}{2}}\cdot\sqrt{\frac{C_{fr}}{2}}\cdot\frac{I_1}{I_2}\\&=-\frac{1}{2}\cdot\frac{\Delta U}{u_o}\frac{I_1}{I_2}\sqrt{\frac{C_{fs}}{C_{fr}}}\end{aligned} \tag{6}$$

For each individual computational case the roughness function must be known a priori. This is normally a mater of importing empirical information(1981) in the likely form of:

$$\frac{\Delta U}{u_o}=2.44\ B_1\ ln\left(\frac{hu_o}{\nu}\right)+B_2. \tag{7}$$

where B_1 and B_2 are experimental constants depending on the characteristic of the surface roughness in question and h is the surface roughness height or another roughness geometry parameter. Of course, more general and flexible forms of Eq. (7) exist. Many excellent papers by Das(1986), Grigson (1987), and Granville(1982) along with the fundamental work of Townsin and his colleagues at Newcastle University(U. K.) are available and these can be traced through Kutlar and Lewkowicz(1990). And the fundamental equation of lag-entrainment method by Das and Lewkowicz(1986) are as follows;

$$\begin{aligned}\frac{dE}{dx}=&\frac{E(E+0.02)+0.267C_{fo}}{\theta(J+H)(E+0.01)}[-2.8((0.32C_{fo}\\&+0.024\mathrm{Eeq}+1.2\mathrm{Eeq}^2)^{1/2}-(0.32C_{fo}+0.024E\\&+1.2E^2)^{1/2}+\left(\frac{\delta}{U_\infty}\frac{dU_\infty}{dx}\right)_{ep}-\left(\frac{\delta}{U_\infty}\cdot\frac{dU_\infty}{dx}\right)]\end{aligned} \tag{8}$$

3. THE OKUNO AND LEWKOWICZ PROGRAM(VERSION ONE, V.I)

Okuno(1976) initiated an integral technique to predict TBLs on slender ship-hulls and the equations are listed in Appendix A. The scheme utilises the well established Hess and Smith(1967) panel method to obtain first the external free-stream potential flow and then to proceed to calculate the boundary layer flow using a boundary layer integral equation approach comprising: (i) the streamwise and crosswise momentum conservation laws, the simple streamwise entrainment principle along with the corsswise moment of momentum conservation law as the flow governing equations, (ii) the power law for streamwise mean velocity profile (with shape factor dependent exponent), (iii) the Okuno (1976) crosswise mean velocity profile, and (iv) the Ludwieg and Tillmann wall friction relationship.

The original Okuno program was subsequently slightly modified by Okuno and Lewkowicz(1984, Version One, V.I) to allow the effects of surface roughness to be explicitly represented in the computations. Okuno and Lewkowicz (1984) outlined the modifications whereby the wall friction relationship applied was that resulting from the Coles law of the wall and of the wake inclusive of the Clauser roughness function (downward velocity shift), $\Delta U/u_o$.

In this form the program was applied to a slender hull model shown in Fig. 1 which was tested experimentally in a wind-tunnel at Liverpool University. The test results provided a data bank of the wall friction coefficient values on the hull in a smooth and artificially roughened (sand grain) state. Okuno and Lewkowicz (1987) and Okuno, Lewkowicz and Nicholson (1985) published the experimental results and compared them with their computational counterparts obtained by the above mentioned scheme (Version V.I). As can be seen from Figs. 5 and 7 of Okuno and Lewkowicz (1987), the agreement between the experimental and computational results for friction coefficients was quite good but not perfect and the present authors sought to improve the program further.

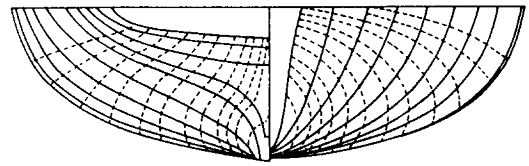

Fig. 1 Body plan and potential streamlines of model.

Other experimental C_f-values for the particular hull form were available from extensive sea trials completed within the research programme described by Nicholson & Lewkwowicz (1983). These trials were backed by adjunct pipe flow tests, described in detail by Musker, Lewkowicz & Preston (1976); they provided the required Clauser roughness function, $\Delta U/u_o$. An elaborate statistical roughness topography and geometry analysis was also carried out.

4. PRESENT MODIFICATIONS TO THE VERSION V.I

The program itself was kept basically intact in its lay-out and structure but improvements(Version Two, V.Ⅱ) were incorporated with auxiliary shape parameter and lag-entrainment equation in velocity profile distribution for turbulent boundary layer and in Head's entrainment equation (1956) to account more effectively for surface roughness effects by;

(a) Making the auxiliary shape parameter for H(= streamwise displacement thickness to streamwise momentum thickness ratio) more responsive to the roughness effects upon the wall friction which was a computerised version V.I of the concept evolved in Sec. 2 above.

(b) Corporating the streamwise entrainment equation with its more powerful and upgraded lag-entrainment in version V. I; no adjustments whatsoever were made to the Green et al. (1973) and modified by Das and Lewkowicz(1986) as auxiliary relationships and/or constants pertaining to the lag-entrainment scheme.

Thus altered program (Version Two, V.Ⅱ) which was modified by version V.I runs well for all tried flow cases free from any numerical peculiarities nor significant increases in the CPU time(typically 7 s on IBM 3081).

To assess separately the influence of the two effects (a) and (b) above, it was possible to 'switch' them on and off within the V.II program at wall. Some of the consequences will be lighlighted later.

5. COMPUTATIONS USING THE MODIFIED PROGRAM (V.II)

Calculations were carried out first for the smooth and fully rough hull model at hull Reynolds number $Re = L_{pp}\ V/\nu = 1.38\times10^6$, where V = undisturbed upstream velocity (ship speed), and then for the geometrically similar full size hull for which sea trials had been performed, as reported by Okuno, Lewkowicz & Nicholson (1985) and by Nicholson and Lewkowicz (1983), at ship $Re = 9.46\times10^8$. For both rough hulls (model and full size ships), the Clauser roughness function was, of course, known in advance following the usual pipe flow tests, e.g. Musker et al. (1974). For the model and the ship, the roughness functions were ;

$$\frac{\Delta U}{u_o} = -3.76X^3 + 17.93X^2 - 20.38X + 7.53, \quad \text{for } 0.8 < X < 2.0 \tag{9a}$$

$$\frac{\Delta U}{u_o} = 1.56X^2 - 2.74X + 1.25, \quad \text{for } 1.0 < X < 2.4 \tag{9b}$$

respectively, where $X = \log(hu_o/\nu)$ and h is the roughness height ($h_s = 290\ \mu m$ for model and $h_{MAA} = 338\mu m$ for ship). The above relationships are polynomial approximations to the experimental $\Delta U/u_o$ data.

Additional calculations with the modified program were performed for the smooth model hull with the roughness subroutine void but including the lag-entrainment changes.

Prior to the presently described modifications, the original Okuno program was also run to ascertain the formerly published results and these were duly confirmed.

First, the outcome of the present computations is shown in Fig.2 where the values of the streamwise momentum thickness, θ_{11} for the model, at probably the most representative WL3, which means waterline 3, are displayed for five different computational cases : (i) smooth hull computed by V.I, (ii) smooth hull computed by V.II, (iii) fully rough hull computed by V.II but with the ΔH-effect (represented by Eq. (6)) switched off, (iv) fully rough hull by V.II with ΔH effect included, and (v) fully rough hull computed by V.I for comparisons. Interestingly, only very little effect on θ_{11} can be seen on either smooth or rough hull, if the lag-entrainment modification alone is applied, however the ΔH effect is quite noticeable.

The basic streamwise shape parameter, $H = \theta_{11}/\delta^*_1$, along WL3 is plotted in Fig. 3 using the two computational versions ; it contains five curves for the same cases as in Fig. 2. Again, it can be seen that the lag-entrainment modification alone imposes a much weaker influence on the parameter than the ΔH correction, although it does reduce slightly H values on the smooth hull. As expected, a strong effect of the surface roughness on H is expectedly conveyed by the ΔH-modification. What must be realised is that the TBL along WL3 is largely of a flat plate nature (typical to midships). The lag-entrainment comes to fore in pressure gradient TBL flows where the upstream history is of importance.

Figure 4 demonstrates the development of the limiting wall

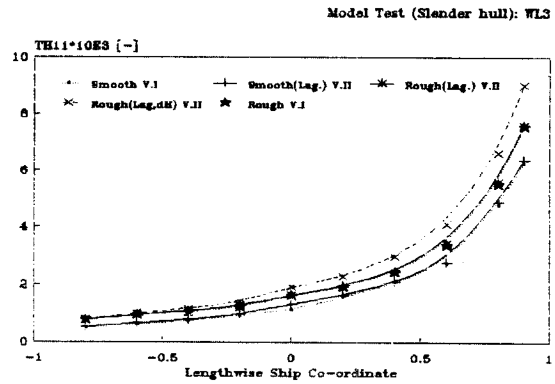

Fig. 2 Streamline momentum thickness

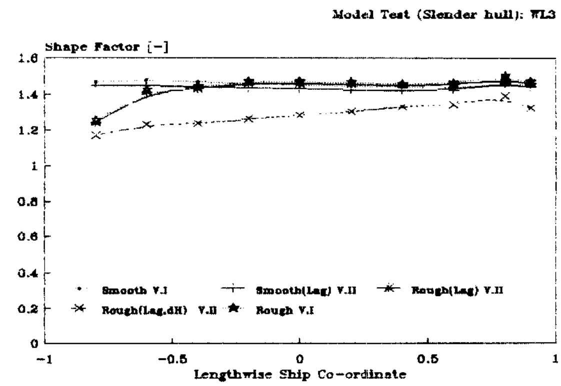

Fig. 3 Streamwise shape factor

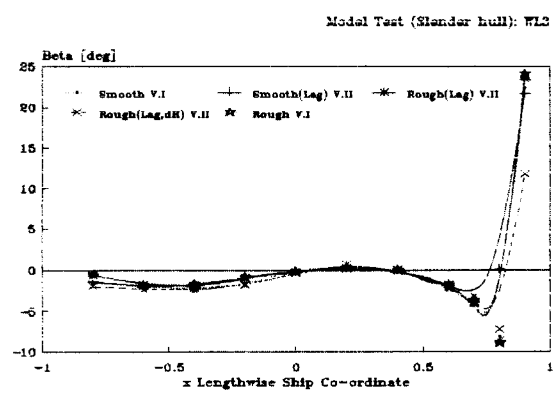

Fig. 4 Surface flow angle

cross-flow angle β_o, as invented by Okuno (1976), along WL3 :

$$\frac{U_1}{U_2} = (\tan\beta_o + C\eta)(1-\eta)^2 \tag{10}$$

where, $\eta = x_2/\delta$ = B.L. transverse co-ordinate, δ = BL absolute thickness, C = proportionality parameter in the cross-flow model, and subscripts 1 and 2 denote streamwise and corsswise components, respectively. For almost the entire length of hull (except stern), the value of β_o is in a range $-5° < \beta_o < 0°$ indication, as expected, a weak three-dimensional TBL.

All computational variants more or less mutually agree throughout the hull and there is minimal surface roughness influence on β_o. Nearer to aft some differences appear: the lag-entrainment seems to change things somewhat but it is not easy to discern any systematic trend. At stern, on approach to separation, β_o rapidly grows to values greater than $+20^\circ$.

Of main interest in this exercise was to calculate the wall friction along WL3, which means water line 3, of the slender hull and compare the predictions with measurements published by Okuno and Lewkowicz (1987). These results were assembled together in Fig. 5. It is apparent that the lag-entrainment modification is beneficial to the predictions on smooth and rough hulls alike, although morestrongly so for the latter, especially, when combined with ΔH-correction. In fact V.II reproduces the experimental data on the rough hull very well. It constitutes a significant improvement over V.I.

Data utilised in Fig. 5 were further processed to obtain the girth averaged values and as such are given in Fig. 6. Here also it is clear that V.II yields quite encouraging agreement with most of the experiments.

The wall friction distribution around the hull allows to calculate the friction resistance of the hull (at Froude number=0) by integrating the C_f over its wetted surface (between WL0 and LWL which means zero and load water line respectively). This calculation is only approximate as it is assumed that past separation the wall friction was zero and of course that the presesnt method predicts separation correctly. It transpires from the model test experimental data that the surface roughness accounts for epf$C_f = 1.31 \times 10^3$ increase in the total hull friction coefficient over that for the smooth hull. Fig. 7 shows that V.I underestimates the increase by some 19% whereas V.II does so by 83% (!). Agin, V.II performed markedly better than V.I as regards the smooth hull predictions.

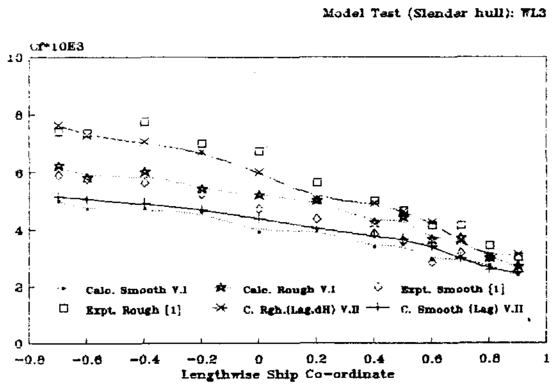

Fig. 5 Local wall friction coefficient

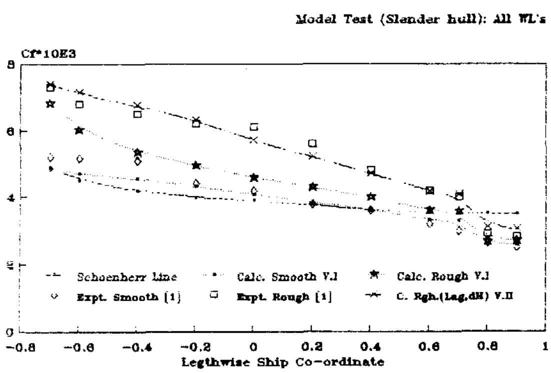

Fig. 6 Girth averaged wall friction coefficient

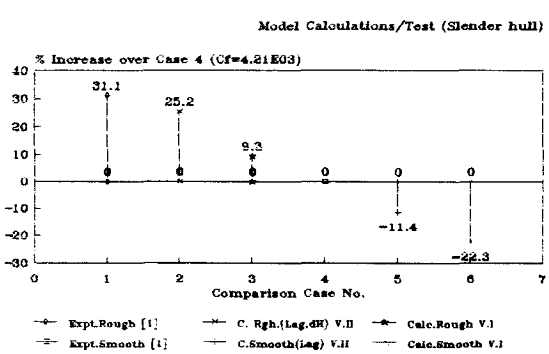

Fig. 7 Roughness caused increase in hull friction resistance (% over smooth).

6. ZONALLY ROUGH HULL

Policy decisions regarding the economy and effectiveness of hull cleaning and antifouling protection may be made on the basis of the idea of 'zonal' hull treatment. It had been considered in the past. Notably, Baba and Tokunaga (1980), Kauczynski and Walderhnug (1987) as well as Okuno and Lewkowicz (1987) addressed this problem. Zonal analysis of the hull roughness is usually executed by assigning a state of surface roughness to selected quarters of the hull where each quarter is LWL/4 long.

It was convenient to apply the present version V.II in this manner in order to assess how zonal roughening affects the girth averaged C_f along the slender hull. The final output of the exercise, shown in Fig. 8, seems to suggest that the effect diminishes quite markedly if the 'rough' quarter moves progressively aft. Of course, this exercise must be regarded with caution since no proper account would be made of the consequences of zonal roughening upon the separated flow at the stern. Nevertheless, the results could serve as a handy indicator of a prevailing trend.

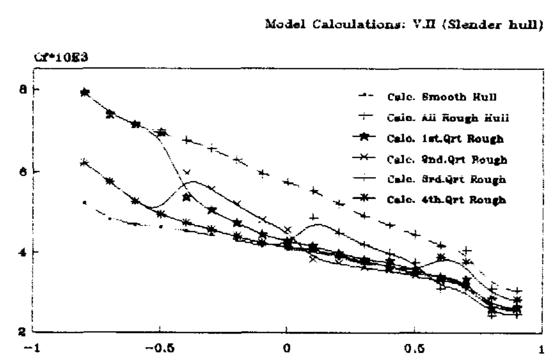

Fig. 8 Effect of zonal hull roughness on girth averaged wall friction coefftcient

7. COMPARISONS WITH SEA TRIALS AND MODEL-TO-SHIP SCALING

Now the new version V.II was used to recalculate the TBL on a full size ship whose slender hull and the same shape as the tested model. Link is here made with the sea trials referred by by Nicholson and Lewkowicz(1983), Okuno. Lewkowicz and Nicholson (1985) and Okuno and Lewkowicz (1987). For those measurements taken at sea the present calculations offer a much improved, general agreement compared with V.I. Combined comparisons, including the Schoenherr reference line, are assembled in Fig. 9. The reasons why V.I yields much poorer results (beyond hitherto encountered differences) were not investigated in depth but are thought to have been mainly due to the much higher Reynolds number for the full size hull. At those conditions the lag-entrainment relationship as a governing equation could steer better the overall TBL solution. For the same reason the ΔH -correction might also be more effective.

Comparisons of girth averaged values of C_f computed for the model and the full size ship are shown in Fig. 10 in which the effects of hull roughness along with the inadequacy of the Schoenherr equation are highlighted.

Finally, when testing hull models and transferring the laboratory data to the full size counterpart, the correct technique of scaling becomes to be some practical importance. Sasajima and Himeno(1965) proposed a simple formula linking the surface roughness induced increase in the hull friction coefficient for the model with that for the full size ship. The formula reads:

$$\frac{\Delta C_{fs}}{\Delta C_{fm}}=\left(\frac{C_{fs}}{C_{fm}}\right)^n \tag{11}$$

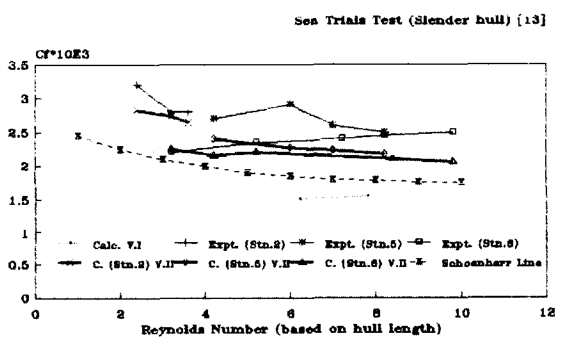

Fig. 9 Comparison of sea trials results with present calculations

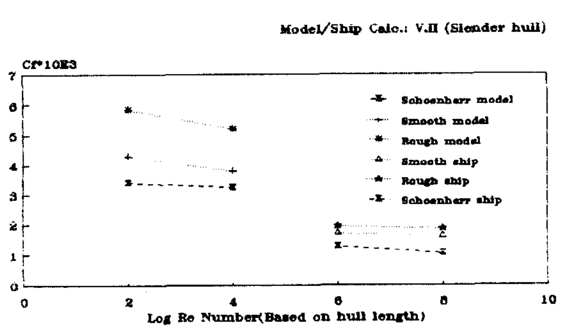

Fig. 10 Model-to-ship scaling : grth averaged wall friction coefficient

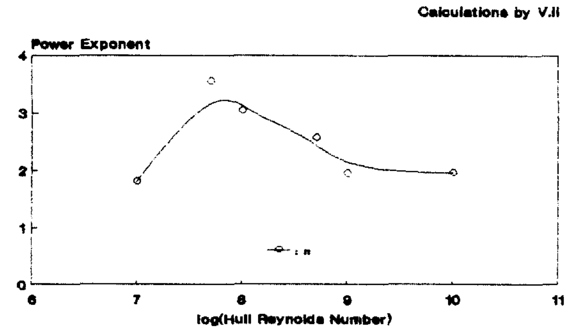

Fig. 11 Power exponent in Sasajima-Himeno scale equation

which is valid if the roughness Reynolds number $(U_\infty h/\nu)<10^3$ is similar for the model and the ship. It was found that $n=2$ yields the best correlation for Eq. (11). The formula was checked experimentally by Baba & Tokunaga (1980) who tested it on two large hull models of similar roughness Reynolds number $\frac{U_\infty h}{\nu}=410$ but of different size.

Their hull Re did not change much, i.e. $2.2\times10^6<R_e<9.3\times10^6$. Watanabe et al. (1969) applied the formula to a slime covered model with $150<(R_e=U_\infty\ \ h/\nu)<760$ and $4.5\times01^7<R_e<7.6\times10^7$ in order to estimate the roughness friction resistance on a full size ship ('Lucy Ashton') for which $1.52\times10^9<\mathrm{Re}<2.53\times10^9$, but could not actually verify the overall validity of the formula.

The present authors used V.II to check if the Sasajima-Himeno formula agrees with the Liverpool University model tests and those for a corresponding full size ship. For the two comparison cases the same roughness function-that for the model tests-was arbitrarily assumed and the roughness Reynolds number set at $U_\infty\ h/\nu=400$ for the ship and the model. Computations were performed for a series of ship Reynolds numbers: $10^7<Re<10^{10}$. Now, $n=n(Re)$ was ab initio anticipated and Fig. 11 gives evidence to this. Whilst $n=2$ (Sasajima and Himeno (1965) holds around $Re=10^7$ and for $RMe>10^9$, the present calculations seem to reveal that for $1.1\times10^7<Re<10^9$ n tends to have significantly higher values reaching even $n\fallingdotseq3.5$.

8. CONCLUSIONS

(1) Further modification has been implemented to the Okuno and Lewkowicz method(1987) for predicting slender ship-hull turbulent boundary layers aiming at a more realistic representation of the hull roughness effects.

(2) The modifications proved to be quite successful in calculating more accurately the wall friction resistance coefficient on a smooth and arbitrarily roughened slender hull model compared with wind-tunnel tests.

(3) It was demonstrated that, disregarding the complex effect upon the turbulent boundary layer transition for zonally roughened hull, the most important contribution to the overall hull frictional resistance coefficient comes from the first quarter (counting from bow) of the hull wetted area. The contribution progressively diminishes as successive hull quarter areas fore to aft are individually considered. This analysis is unable to take into account the role of stern separation and other thereto related effects.

(4) An important model-to-ship scaling formula was

devised(1965) making it possible to estimate approximately the roughness correction to the hull frictional resistance coefficient of a full size ship hull from its geometrically similar roughened model counterpart. The validity of this simple formula was checked and showned that, although valid in its original form for some hull Reynolds numbers, it may need to be adjusted if $1.1\times10^7 < Re < 10^9$.

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REFERENCE

Baba, E. and Tokunaga, K.,1981, "Study of Local Roughness Effects on Ship Resistance for Effective Cleaning and Protection of Hull Surface, Proc. SNAME ship Board Energy Conservation, '80. Smooth to Rough," M315-30 Vol. 48, pp. 721 ~762.

Granville, P.S., 1982, "Drag Characterizataion Method for Arbitarily Rough Surface by Means of Rotary Disc," J.of Fluids Engineering, Vol. 104, pp 373~377.

Green, J.E., Weeks, D.J. and Brooman, J.W.F., 1977, "Prediction of Turbulent Boundarys layers and Wakes Incompressible Flow by a Lag-interainment Method," Royal Aircraft Establishment, R & M No. 3791.

Head,H.R., 1956, "Entrainment in the Turbulent Boundary Layer," Aeron. Coun., R & M 3152.

Grigson, C., 1981, "The Full-scale Viscous Drag of Actual Ship Surface and the Effects of Quality of Roughness on Predicted Power," J. of Ship Research, Vol.31, pp 189~206.

Heokstra, M.,1983, "Incorperation of Wall Roughness of Effects in Boundary Layer Calculation," Maritime Reserch Institute Netherlands, MARIN Order No. 50303.

Hess, J.L. and Smith, A.M.O., 1966, "Calculation of Potential Flow About Arbitary Bodies," Prog. Aeron. Soci., Vol. 8.

Kauczynski, W. and Walderhung, H.,1986, "Effect of Paint Roughness on the Skin Friction of Ships," Int'l Ship Building Progress, Vol.34, pp.625~630.

Kutlar, H. and Lewkowicz, A. K., 1990, "Investigation of the Effect of a Paint Coat on the Turbulence Near a Rough Flat Plate," Int'l Workshop on Marine Roughness & Drag, R. I. N. A., London.

Lewkowicz, A.K., 1982, "An Improved Universal wake Function for Turbulent Boundary Layers and Some of it's Consequene," Z. Flugwiss Weltraumforsch, 6 Heft 4.

Lewkowicz, A.K., 1985, "Assessment of the Hydrodynamic Effects of Fouling Ship-hulls," Osaka Int'l Coll. on Ship Viscous Resistance Fluid.

Lewkowicz, A.K.and Das, D.K.,1981, "Turbulent Boundary Layers on Roughness with and Without a Plaiable over Layer ; A simulation of Marin Fouling," Joint ASME/ASCE Bioengineering, Fluid Mechanics and Applied Mechanics, Conf. Bolder, Colorada, U.S.A.

Lewkowicz, A. K. and Das, D.K., 1986, "Turbulent Boundary Layers on Roughness with and Without Plaiable Over Layer," Int'l Shipbuilding Prog, Vol. 33, pp.174~186.

Musker, A. J., Lewkowicz, A.K. and Preston, J. H.,1976, "Investigation of the Effects of Surface Roughness of Ship Wall Friction Using a Pipe Flow Techneque," Rep. FM/24/76, Mech. Engrg. Rept. Liverpool Univ.

Nagato, M., Onogi, H., Himeno, Y., Tanaka, I., and Suzuki, T.,1985, "Resistance Increase Due to Surface Roughness," Proc. 15th Symp. Naval Hydron., Nat'l. Academy Press, Washington D. C., pp. 553~568.

Nicolson, K. and Lewkowicz, A. K., 1983, "Research into the Relationship Between Roughness and Fouling," Proc. Int'l Symp. Ship Hydrodynamics and Engineering Saving, El Pardo, Sept., pp.II. 7-1-10.

Okuno, T.,1976, "Distribution of Wall Shear Stress and Cross Flow in the three Dimensional Turbulent Boundary on Ship Hull," J.S.N.A., Vol. 39, pp. 10~23.

Okuno, T. and Lewkowicz, A. K., 1984, "Users Guide to Computer Program for Turbulent Boundary Layer on Ship-hull," Rep. FM/88/84, Mech. Dept. Liverpool Univ.

Okuno, and Lewkowicz, A. K.,1987, "Wind Tunnel Tests on an Artificially Roughened Ship-hull Model," R.I.N.A., Vol. 129, pp.161~170.

Okuno, T., Lewkowicz, A. K. and Nicolson, K., 1985, "Roughness Effects on Slender Ship Hull," Second In'l. Symp. on Ship Viscous Resistance, SSPA. Goteborg, Sweden, pp. 20 ~21.

Sasajima, H. and Himeno, Y.,1965, "Scale Correction for Roughness Effect," J. Zosen Kiokai, Vol. 118, pp. 66~70.

Watanable, S. et al., 1969, "The Augumentation in Frictional Resistance Due to Slime," J. of the Kan-Sai Soci. N-A. of Japan, Vol. 131, pp. 45~69.

Appendix A

(1) Streamwise momentum

$$\frac{\partial}{h_1\partial\zeta}(U_\infty^2\theta_{11}) + \delta_1 U_\infty\left(\frac{\partial}{h_1}\frac{U_\infty}{\partial\zeta}\right) - K_1 U_\infty^2\theta_{11} = \frac{\tau_0\zeta}{\rho} \quad \text{(A.1)}$$

(2) Crosswise Momentum

$$\frac{\partial}{h_1\partial\zeta}(U_\infty^2\theta_{21}) + 2K_1 U_\infty^2\theta_{21} + K_2 U_\infty^2(\theta_{11}+\delta_1) = \frac{\tau_0\eta}{\rho} \quad \text{(A.2)}$$

(3) Streamwise boundary layer entrainment

$$\frac{\partial}{h_1\partial\zeta}(\theta_{11}E_E) + \left(\frac{1.0}{U_\infty h_1}\cdot\frac{\partial U_\infty}{\partial\zeta} - k_1\right) = F(H_E) \quad \text{(A.3)}$$

where,

$$F(H_E) = 0.0306(H_E - 3.0)^{-0.653},$$
$$H_E = 1.535(H - 0.7)^{-2.175} + 3.3$$

(4) Crosswise moment of momentum

$$\frac{\partial}{h_1\partial\zeta}(U_\infty^2 M_{21}) + 2K_1 U_\infty^2(M_{11}+M_1) + U_\infty^2 Q = 0. \quad \text{(A.4)}$$